Study Guide for Math 095

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1 The Real Number System

Writing a fraction in lowest terms.

1. Find the largest number that will divide into both the numerator and the denominator. This number is called the **greatest common factor** (GCF).

2. Divide both the numerator and the denominator by the GCF.

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

Multiplying fractions.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \qquad \frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$$

Dividing fractions. To divide two fractions, invert the second fraction and multiply.

$$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \cdot \frac{7}{3} = \frac{14}{15}$$

$$\frac{5}{6} \div 3 = \frac{5}{6} \div \frac{3}{1} = \frac{5}{6} \cdot \frac{1}{3} = \frac{5}{18}$$

Adding fractions with the same denominators.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \qquad \frac{2}{7}+\frac{4}{7}=\frac{6}{7}.$$

Adding fractions with different denominators.

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \qquad \frac{2}{9}+\frac{3}{4}=\frac{2\cdot 4+9\cdot 3}{9\cdot 4}=\frac{8+27}{36}=\frac{35}{36}.$$

Subtracting fractions with the same denominators.

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b} \qquad \frac{7}{10}-\frac{6}{10}=\frac{1}{10}.$$

Subtracting fractions with different denominators.

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \qquad \frac{3}{4}-\frac{2}{5}=\frac{3\cdot 5-4\cdot 2}{4\cdot 5}=\frac{15-8}{20}=\frac{7}{20}.$$

Order of operations. If parentheses or fraction bars are present, simplify within parentheses, innermost first, and above and below fraction bars separately, in the following order.

1. Apply all exponents.
2. Do any multiplications or divisions in the order in which they occur, working from left to right.
3. Do any additions or subtractions in the order in which they occur, working from left to right.

If no parentheses or fraction bars are present, start with Step 1.

Example:

$$\begin{aligned} 7^2-\frac{(6+4\cdot 5)}{2} &= 7^2-\frac{6+20}{2} \\ &= 7^2-\frac{26}{2} \\ &= 49-13 \\ &= 36. \end{aligned}$$

Additive inverse. "Additive inverse" means "negative." The additive inverse of 4 is -4, the additive inverse of -3 is 3, etc. The additive inverse of 0 is 0. When a number and its additive inverse are added, the sum is 0.

Double negative rule. $-(-x) = x$. (Two wrongs make a right.)

Absolute value. The absolute value of x is denoted by $|x|$. If x is a positive number or zero, then its absolute value is x itself. However, if x is a negative number, then its absolute value is $-x$. The absolute value of a number is never negative!

Examples: $|5| = 5$, $|-3| = 3$, $|0| = 0$.

Adding signed numbers.

> **Same signs:** Add the absolute values of the numbers. The sum has the same sign as the given numbers.
>
> **Opposite signs:** Find the difference of the larger absolute value and the smaller. Give the answer the sign of the number having the larger absolute value.

Examples: $(-4)+(-6) = (-10)$ $\quad(-7)+(+3) = (-4)$ $\quad(-4)+(+9) = (+5)$

Definition of subtraction. $x - y = x + (-y)$.

Subtracting signed numbers. Change the subtraction symbol to addition, change the sign of the number being subtracted, and add.

Example: $(-4) - (-2) = (-4) + (+2) = (-2)$.

Multiplication by zero. $x \cdot 0 = 0$. Anything times zero is zero.

Division by zero is undefined! For example, $6 \div 0$ is undefined. On the other hand, $0 \div 6 = 0$.

Multiplying and dividing signed numbers. The product or quotient of two numbers having the **same** sign is **positive**. The product or quotient of two numbers having **opposite** signs is **negative**.

Multiplicative inverses. A pair of numbers are multiplicative inverses (or reciprocals) of each other if their product is 1. The reciprocal of x is $\frac{1}{x}$, and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Axioms of addition and multiplication.

	Addition	Multiplication
Commutative	$x + y = y + x$	$xy = yx$
Associative	$(x + y) + z = x + (y + z)$	$(xy)z = x(yz)$
Identity	$x + 0 = x$	$x \cdot 1 = x$
Inverse	$x + (-x) = 0$	$x \cdot \frac{1}{x} = 1 \quad (x \neq 0)$
Distributive	$x(y + z) = xy + xz$	

2 Solving Equations and Inequalities

Addition property of equality. The same quantity may be added to (or subtracted from) both sides of an equation. For example, $2x - 4 = 3$ and $2x - 4 + 4 = 3 + 4$ have the same solution.

Multiplication property of equality. Both sides of an equation may be multiplied or divided by the same nonzero number.

Solving a linear equation.

1. Multiply both sides by the least common denominator (LCD) in order to remove fractions.
2. Use the distributive law to remove all parentheses.
3. Move the variable terms (such as $2x$) to one side of the equation, and constant terms (such as -6) to the other side, by using the Addition Property of Equality.
4. Combine like terms on each side of the equation.
5. Divide both sides by the coefficient of the variable.
6. Check the solution by substituting into the original equation.

Example:

$$\begin{aligned}
(1/2)x + 3(2x-1) &= 14 - 2x \\
x + 6(2x-1) &= 28 - 4x && (1)\\
x + 12x - 6 &= 28 - 4x && (2)\\
x + 12x + 4x &= 28 + 6 && (3)\\
17x &= 34 && (4)\\
x &= 34/17 = 2 && (5)\\
\textit{Check:} \quad (1/2)\cdot 2 + 3(2\cdot 2 - 1) &= 14 - 2\cdot 2 && (6)\\
1 + 3(3) &= 14 - 4 \\
10 &= 10 \quad \surd
\end{aligned}$$

Solving word problems.

1. Read the problem carefully. Determine which information is given to you in the problem, and which information you are asked to find.
2. Choose a variable to represent the unknown quantity. State explicitly what the variable represents (e.g. **Let x = number of dimes**).
3. Translate the problem into an equation. Drawing a picture is often helpful.
4. Solve the equation.
5. Answer the question asked in the problem!
6. Check your solution by using the original words of the problem.

Translating key words and phrases.

Key words and phrases	Verbal description	Algebraic Statement
Equality		
Equals, equals to, is, are, was, will be, represents	The sales price S is \$10 less than the list price L.	$S = L - 10$
Addition		
Sum, plus, greater than,	The sum of 5 and x	$5 + x$
increased by, more than,	Seven more than y	$y + 7$
exceeds, total of	x exceeds y by 2	$x = y + 2$

Subtraction		
Difference, subtracted from	The difference of 4 and b	$4 - b$
less, minus, decreased by,	Three less than z	$z - 3$
reduced by, the remainder	The width reduced by 1	$w - 1$
Multiplication		
Product, multiplied by,	Two times x	$2x$
twice, times, percent of	Five percent of x	$.05x$
Division		
Quotient, divided by,	The quotient of x and 8	$\frac{x}{8}$
ratio, per	The ratio of x to y	x/y

Ratio and proportion. A **ratio** is a quotient of two quantities. They are often expressed using the word "per." (Miles per gallon, ten percent, etc.) Ratios may be expressed in many ways, including:

$$a \text{ to } b, \qquad a : b, \qquad a/b, \quad \text{and} \quad \frac{a}{b}.$$

A **proportion** is a statement that two ratios are equal.

Cross multiplication. $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$. For example, to solve $\frac{6}{x} = \frac{4}{3}$, we could cross multiply to obtain $6 \cdot 3 = x \cdot 4$.

Addition property of inequality. The same quantity may be added to (or subtracted from) both sides of an inequality. For example, the inequalities $x < y$ and $x + 3 < y + 3$ are equivalent.

Multiplication property of inequality. Both sides of an inequality may be multiplied or divided by a positive number. When you multiply or divide by a negative number, the direction of the inequality is reversed.

For example, $2x < -8$ is equivalent to $x < -4$, but $-2x < 8$ is equivalent to $x > -4$. In the second case, the '$<$' switches to a '$>$' because we are dividing by -2.

Solving an inequality. You solve an inequality in exactly the same way that you solve an equation, with only two exceptions:

1. If you multiply or divide both sides by a negative number, you must reverse the inequality.
2. If you exchange the sides of an inequality, you must reverse the inequality (for example, $2 \leq x$ becomes $x \geq 2$).

Note that adding and subtracting will never reverse the direction of an inequality.

Example: Solve $2 - 3x \leq 10 + x$
Solution:

$$\begin{aligned} 2 - 3x &\leq 10 + x \\ -3x - x &\leq 10 - 2 \\ -4x &\leq 8 \\ x &\geq -2 \end{aligned}$$

3 Polynomials and Exponents

Adding polynomials. To add two polynomials, add like terms.

Subtracting polynomials. To subtract two polynomials, change all the signs on the second polynomial and add the two polynomials.

$$\begin{aligned} (x^2 - 4x) - (3x^2 - 9x) &= (x^2 - 4x) + (-3x^2 + 9x) \\ &= (1 - 3)x^2 + (-4 + 9)x \\ &= -2x^2 + 5x \end{aligned}$$

Negative base of exponents.

$$(-a)^n = \begin{cases} a^n & \text{if } n \text{ is even,} \\ -a^n & \text{if } n \text{ is odd.} \end{cases}$$

Rules of exponents.

Product rule	$a^m \cdot a^n = a^{m+n}$
Zero exponent	$a^0 = 1$
Negative exponent	$a^{-n} = \dfrac{1}{a^n}, \quad \left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$
Quotient rule	$\dfrac{a^m}{a^n} = a^{m-n}$
Power of a product	$(ab)^n = a^n b^n$
Power of a quotient	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
Power to a power	$(a^m)^n = a^{mn}$

Multiplying polynomials. Multiply each term of the second polynomial by every term of the first polynomial, and add the products. When each polynomial has only two terms, this reduces to the **FOIL method**. FOIL is an abbreviation for *first, outer, inner, last.*

$$\begin{aligned}(2x+3)(4x+y) &= 2x(4x) + 2x(y) + 3(4x) + 3(y)\\ &= 8x^2 + 2xy + 12x + 3y\end{aligned}$$

Square of a binomial.

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Do not forget the middle term! In general, $(x+y)^2 \neq x^2 + y^2$.

Changing from negative to positive exponents. A factor may be moved from the numerator to the denominator, or from the denominator to the numerator, by changing the sign of the exponent.

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}, \qquad \frac{3^{-3}x^6}{4y^{-9}} = \frac{x^6y^9}{4 \cdot 3^3}$$

Dividing a monomial by a monomial. First divide the numerical coefficients, then divide each variable using the quotient rule.

$$\frac{12x^8y^6}{3x^7y} = \left(\frac{12}{3}\right)\left(\frac{x^8}{x^7}\right)\left(\frac{y^6}{y}\right) = 4x^{8-7}y^{6-1} = 4xy^5$$

Dividing a polynomial by a monomial. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example:

$$\begin{aligned}\frac{16a^5 - 12a^4 + 8a^2}{4a^3} &= \frac{16a^5}{4a^3} - \frac{12a^4}{4a^3} + \frac{8a^2}{4a^3} \\ &= 4a^2 - 3a + \frac{2}{a}\end{aligned}$$

Long division. Long division should be used when the divisor has more than one term. The first step is to arrange the term in order of descending powers. Use a coefficient of zero for any missing terms.

Example: Divide $4x^3 + 1 - 5x$ by $2x + 3$.

Solution: $2x^2 - 3x + 2 - \dfrac{5}{2x+3}$

$$\begin{array}{rl} & 2x^2 - 3x + 2 \\ 2x+3 \big) & 4x^3 \qquad\quad - 5x + 1 \\ & -\ 4x^3 - 6x^2 \\ \hline & -\ 6x^2 - 5x \\ & 6x^2 + 9x \\ \hline & \qquad 4x + 1 \\ & \quad -\ 4x - 6 \\ \hline & \qquad\quad -\ 5 \end{array}$$

$$\begin{array}{ccc} 4x^3/2x = 2x^2 & \qquad & 2x^2(2x+3) = 4x^3 + 6x^2 \\ -6x^2/2x = -3x & & -3x(2x+3) = -6x^2 - 9x \\ 4x/2x = 2 & & 2(2x+3) = 4x + 6 \end{array}$$

Note that we change signs before adding.

4 Factoring and Applications

Factoring polynomials

1. *Remove common factors.*
   (a) $6x^3 - 4x = 2x(3x^2) - 2x(2) = 2x(3x^2 - 2)$
   (b) $(4x - 5)(2x) + (4x - 5)(y) = (4x - 5)(2x + y)$
   (c) $10x^2y^3 - 12x^3z^2 = (2x^2)(5y^3) - (2x^2)(6xz^2) = (2x^2)(5y^3 - 6xz^2)$
2. *Check for* **special factorizations.**

Difference of two squares	$x^2 - y^2 = (x - y)(x + y)$
Difference of two cubes	$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Sum of two cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

3. *If there is a trinomial of degree two, use trial and error.*
   (a) *To factor* $x^2 + Ax + B$*, find two numbers whose sum is* A *and whose product is* B.
      i. $x^2 + 9x + 18 = (x + 6)(x + 3)$
      ii. $x^2 - 3x - 40 = (x + 5)(x - 8)$
   (b) *Factoring a trinomial of the form* $Ax^2 + Bx + C$*:*

$$5x^2 + 13x - 6 = (5x - 2)(x + 3)$$

Product of the two firsts	=	First term of trinomial
Product of the two lasts	=	Last term of trinomial
Sum of inner and outer products	=	Middle term of trinomial

4. *If there are four terms, try to* **factor by grouping.**

Step 1 *Write the four terms so that the first two terms have a common factor and the last two terms have a common factor.*

Step 2 *Factor each group of two terms.*

Step 3 *Factor a common binomial factor from the results of Step 2.*

Example: *Factor* $6x^2 + 4xy - 9x - 6y$
Solution:

$$\begin{aligned} 6x^2 + 4xy - 9x - 6y &= (6x^2 + 4xy) - (9x + 6y) \\ &= 2x(3x + 2y) - 3(3x + 2y) \\ &= (2x - 3)(3x + 2y) \end{aligned}$$

Zero factor property. *If the product of two or more numbers is zero, then one of the factors must be equal to zero. In symbols, if* $ab = 0$*, then either* $a = 0$ *or* $b = 0$*.*

Solving quadratic equations by factoring.

1. *Move all terms to one side of the equals sign, leaving 0 on the other side.*

2. *Factor completely.*

3. *Set each factor to 0, and solve the resulting equations.*

Example: *Solve* $2x^2 + 4 = 9x$
Solution:

$$2x^2 - 9x + 4 = 0$$
$$(2x - 1)(x - 4) = 0$$
$$2x - 1 = 0 \quad \textit{or} \quad x - 4 = 0$$
$$x = 1/2,\ 4$$

Pythagorean theorem. *If a triangle has longest side of length* c *and two other sides of lengths* a *and* b*, then*

$$c^2 = a^2 + b^2$$

5 Rational Expressions

A **rational expression** *is the quotient of two polynomials* $\frac{P}{Q}$*. If* $Q = 0$ *then* $\frac{P}{Q}$ *is* **undefined***. To determine when a rational expression is undefined, set the denominator equal to zero and solve.*

Reducing to lowest terms. *To write a rational expression in lowest terms, (1) factor the numerator and denominator, and (2) cancel common factors from the numerator and denominator.*

$$\frac{x^2 - 5x + 6}{x^2 - 4} = \frac{(x-2)(x-3)}{(x-2)(x+2)} = \frac{x-3}{x+2}$$

Multiplication of rational expressions.

1. *Factor completely.*
2. *Multiply numerators and multiply denominators.*
3. *Write in lowest terms.*

   To divide two fractions, invert the second fraction and multiply.

Finding the least common denominator.

1. *Completely factor all denominators.*
2. *Take each different factor the* greatest *number of times that it appears as a factor in any of the denominators.*
3. *The least common denominator is the product of all factors to the greatest power found in step 2.*

Adding fractions with different denominators.

1. *Find the least common denominator (LCD).*
2. *Rewrite each rational expression as a fraction with the LCD as the denominator.*

3. *Add the numerators to get the numerator of the sum. The least common denominator is the denominator of the sum.*

4. *Reduce, if possible.*

Simplifying complex fractions. *Find the least common denominator of* all *fractions appearing in the complex fraction. Multiply the numerator and denominator by this LCD.*

$$\frac{\frac{5}{8}+\frac{2}{3}}{\frac{7}{3}-\frac{1}{4}} = \frac{\frac{5}{8}\cdot 24+\frac{2}{3}\cdot 24}{\frac{7}{3}\cdot 24-\frac{1}{4}\cdot 24} = \frac{15+16}{56-6} = \frac{31}{50}$$

Solving equations with rational expressions.

1. *Find the least common denominator.*

2. *Multiply both sides by the LCD in order to clear fractions.*

3. *Solve the resulting equation.*

4. *Reject any solutions which make a denominator equal to 0. These solutions are called* **extraneous**.

Example: *Solve* $\frac{1}{x} + \frac{2}{x+2} = \frac{4x-9}{x^2+2x}$

Solution:

$$\frac{1}{x} + \frac{2}{x+2} = \frac{4x-9}{x(x+2)}$$

$$\frac{1}{x}x(x+2) + \frac{2}{x+2}x(x+2) = \frac{4x-9}{x(x+2)}x(x+2)$$

$$(x+2) + 2x = 4x - 9$$

$$3x + 2 = 4x - 9$$

$$3x - 4x = -2 - 9$$

$$-x = -11$$

$$x = 11$$

6 Graphing Linear Equations

Finding intercepts of a line. *To find the x-intercept of a line, set $y = 0$ in the given equation and solve for x. To find the y-intercept of a line, set $x = 0$ in the given equation and solve for y.*

Horizontal and vertical lines. *The equation of a horizontal line is of the form $y = k$, and the equation of a vertical line is of the form $x = k$. The slope of a horizontal line is zero, and the slope of a vertical line is* **undefined**.

Slope of a line.

$$\text{slope} = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, the slope of the line through $(1,5)$ and $(4,2)$ is $(2-5) \div (4-1) = -1$.

To find the slope of a line from its equation, solve for y. The slope is the coefficient of x.

Slope–intercept form. *When the equation of a line is in the form*

$$y = mx + b,$$

the slope of the line is m and the $y-$intercept is $(0, b)$. For example, the line $y = -3x - 7$ has a slope of -3 and a y-intercept of $(0, -7)$.

Point–slope form. *An equation of the line with slope m going through (x_1, y_1) is*

$$y - y_1 = m(x - x_1)$$

Example: *Find the equation of the line through $(2, -3)$ with slope $\frac{1}{2}$.*
Solution: $y - (-3) = \dfrac{1}{2}(x - 2)$

Equation of line through two points. *To find the equation of a line through two given points, find the slope using the formula $m = (y - y_1)/(x - x_1)$. Substitute the slope and the coordinates of either of the two points into the point–slope formula.*

Example: *Find the equation of the line through $(1, 3)$ and $(4, 9)$.*

$$\text{slope} = \frac{9-3}{4-1} = 2$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \qquad m = 2, x_1 = 1, y_1 = 3 \\ y - 3 &= 2(x - 1) \\ y - 3 &= 2x - 2 \\ y &= 2x + 1 \end{aligned}$$

Parallel and perpendicular lines. *Two lines with the same slope are parallel. Two lines that have slopes with a product of* -1 *are perpendicular.*

7 Linear Systems

Solving systems by graphing. *One way to find the solution of a system of two linear equations is to graph the equations on the same axes. The coordinates of the point where the lines intersect give the solution.*

Example: *The lines* $x + y = 8$ *and* $x - y = 2$ *intersect at* $(5, 3)$*, so the solution to this system of equations is* $x = 5$*,* $y = 3$*.*

Inconsistent and dependent equations. *A system of equations is* **inconsistent** *if it has no solution. A system of equations with infinitely many solutions is called* **dependent***.*

Solving systems by addition.

Step 1 *Write both equations in the form* $Ax + By = C$*.*

Step 2 *Multiply one or both equations by appropriate numbers so that the coefficients of* x *(or* y*) are negatives of each other.*

Step 3 *Add the equations to get an equation with only one variable.*

Step 4 *Solve the equation from Step 3.*

Step 5 *Substitute the solution from Step 4 into either of the original equations to find the value of the remaining variable.*

Step 6 *Write the solution as an ordered pair and check the answer.*

Solving systems by substitution.

Step 1 *Solve one of the equations for either variable.*

Step 2 *Substitute for that variable in the other equation. The result should be an equation with just one variable.*

Step 3 *Solve the equation from Step 2.*

Step 4 *Substitute the result from Step 3 into the equation from Step 1 to find the value of the other variable.*

Step 5 *Check the solution in both of the given equations.*

Solving word problems with two variables.

Step 1 *Choose a different variable for each of the two unknown values you are asked to find. Write down what each variable is to represent.*

Step 2 *Translate the problem into two equations using both variables.*

Step 3 *Solve the system of two equations.*

Step 4 *Answer the question or questions asked in the problem.*

Step 5 *Check your solution by using the original words of the problem.*

8 Roots and Radicals

Square roots. *A* **square root** *of a number is one of its two equal factors. For example, 5 is a square root of 25 because* $5 \times 5 = 25$. *Every positive number has two square roots; one is positive, the other negative. The symbol* $\sqrt{x}$ *denotes the* **non-negative** *square root only.*

When we speak of "the" square root of a number, we mean the **non-negative** *square root. Thus, the square root of 4 is 2, not* -2.

Definition of nth root. *Let* a *and* b *be real numbers and let* $n \geq 2$ *be a positive integer. If* $a = b^n$ *then* b *is an* **nth root of a.** *If* $n = 2$, *then the root is a* **square root.** *If* $n = 3$, *then the root is a* **cube root.**

Existence of nth roots. *Let* a *be a real number, and let* $n \geq 2$ *be a positive integer.*

1. *If n is odd, then a has exactly one real nth root, which is written as $\sqrt[n]{a}$. This root has the same sign as a.*

2. *If n is even and a is positive, then a has exactly two nth roots, which are additive inverses of each other. The positive nth root is $\sqrt[n]{a}$ and the negative root is $-\sqrt[n]{a}$.*

3. *If n is even and a is negative, then a does not have an nth root.*

4. *0 has exactly one nth root, namely 0. We write $\sqrt[n]{0} = 0$.*

Properties of Radicals

$$\begin{aligned}
&\sqrt{x} \cdot \sqrt{x} = x \\
&\left(\sqrt[n]{x}\right)^n = x \\
&\sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y} && \text{(Product rule)} \\
&\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} && \text{(Quotient rule)}
\end{aligned}$$

Simplifying radicals

1. *If a radicand is a perfect square, then the square root should be used in place of the radical. For example, $\sqrt{49}$ is simplified by writing 7, and $\sqrt{\frac{169}{9}}$ by writing $\frac{13}{3}$.*

2. *If a radical expression contains products of radicals, the product rule for radicals, $\sqrt{x} \cdot \sqrt{y} = \sqrt{xy}$, should be used to get a single radical. For example, $\sqrt{3} \cdot \sqrt{2}$ is simplified to $\sqrt{6}$, and $\sqrt{5} \cdot \sqrt{x}$ to $\sqrt{5x}$.*

3. *If a radicand has a factor that is a perfect square, the radical should be expressed as the product of the square root of the perfect square and the remaining radical factor. For example, $\sqrt{20}$ is simplified to $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$, and $\sqrt{75}$ to $5\sqrt{3}$.*

4. *Any radical in the denominator should be changed to a rational number. For example, $\frac{5}{\sqrt{3}}$ is rationalized as*

$$\frac{5}{\sqrt{3}} = \frac{5 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{5\sqrt{3}}{3}.$$

5. *If a radical expression contains sums or differences of radicals, the distributive property should be used to combine the terms, if possible. For example,* $3\sqrt{2}+4\sqrt{2}$ *is combined as* $7\sqrt{2}$*, but* $3\sqrt{2}+4\sqrt{3}$ *cannot be further combined.*

Squaring property of equality. *If both sides of a given equation are squared, all solutions of the original equation are* among *the solution of the square equation. Whenever this method is used, it is necessary to check for* **extraneous solutions.**

Solving equations with radicals.

Step 1 *Arrange the terms so that one of the radicals is alone on one side of the equation.*

Step 2 *Square both sides.*

Step 3 *Combine like terms.*

Step 4 *If there is still a term with a radical, repeat Steps 1–3.*

Step 5 *Solve the equation.*

Step 6 *Check all solutions from Step 5 in the original equation.*

Fractional exponents.

$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$
$$a^{-m/n} = \frac{1}{a^{m/n}}$$

9 Quadratic Equations

Quadratic formula. *To solve any quadratic equation of the form*

$$ax^2 + bx + c = 0,$$

substitute the coefficients in the **quadratic formula,** *and evaluate.*

Example. *Solve* $2x^2 - 5x + 1 = 0$ *by using the quadratic formula.*

Solution. $2x^2 - 5x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \qquad a = 2,\ b = -5,\ c = 1$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4}$$

The imaginary unit. $i = \sqrt{-1}$ *and* $i^2 = -1$.

Complex numbers. *A* **complex number** *is a number of the form* $a+bi$*, where* a *and* b *are real numbers. The* **real part** *of the complex number* $a+bi$ *is* a*, and the* **imaginary part** *is* b*.*

Complex numbers allow us to define the square root of any negative number. For example, $\sqrt{-64} = \sqrt{64}\sqrt{-1} = 8i$*.*

Addition and subtraction of complex numbers.

1. *To add complex numbers, add their real parts and add their imaginary parts.*
2. *To subtract complex numbers, change the number following the subtraction sign to its negative, and then add.*

Example: $(2 - 6i) + (7 + 4i) = (2 + 7) + (-6 + 4)i = 9 - 2i$

Multiplication of complex numbers. *Multiplication of complex numbers is performed in the same way as multiplication of polynomials (i.e. using FOIL), except that* i^2 *must be replaced with* -1*.*

Example:

$$\begin{aligned}
(2 + 3i)(7 - 4i) &= 2 \cdot 7 - 2 \cdot 4i + 3i \cdot 7 - 3i \cdot 4i \\
&= 14 - 8i + 21i - 12i^2 \\
&= 14 - 8i + 21i + 12 \\
&= 26 + 13i
\end{aligned}$$

Complex conjugate. *The* conjugate *of a complex number is formed by reversing the sign of its imaginary part. The conjugate of* $a + bi$ *is* $a - bi$*, and the conjugate of* $a - bi$ *is* $a + bi$*.*

The product of a complex number and its conjugate is real. In fact, $(a + bi)(a - bi) = a^2 + b^2$.

Division of complex numbers. *To divide a complex number by a real number, divide the real and the imaginary parts of the complex number by the real number.*

To divide two complex numbers, multiply both the numerator and the denominator by the conjugate of the denominator.

Example:

$$\begin{aligned} \frac{1+2i}{2-3i} &= \frac{1+2i}{2-3i} \cdot \frac{2+3i}{2+3i} \\ &= \frac{2+3i+4i+6i^2}{4+6i-6i-9i^2} \\ &= \frac{2+7i-6}{4+9} \\ &= \frac{-4+7i}{13} \\ &= -\frac{4}{13} + \frac{7}{13}i \end{aligned}$$